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MATH 3: Exam 1

Problem 1. (10 points) Consider the two functions $f(x)$ and $g(x)$ defined by the following tables:

x	2	3	4	5
$f(x)$	12	5	6	6

x	5	6	7	9	12
$g(x)$	1	2	3	6	13

Find the domain and range of the composite function $(g \circ f)(x)$:

Domain $(g \circ f)$ = Domain f = $\{2, 3, 4, 5\}$

Range $f = \{5, 6, 12\}$ and $g(5) = 1$, $g(6) = 2$, $g(12) = 13$

$\Rightarrow$ ~~Domain~~ Range $(g \circ f) = \{1, 2, 13\}$

Problem 2. Let $f(x) = \frac{1}{2}x + 1$.

(a) (3 points) Calculate the x-intercept of $f(x)$.

Solve $0 = f(x) = \frac{1}{2}x + 1 \Rightarrow x = -2$

$\Rightarrow$ x-intercept is $(-2, 0)$

(b) (3 points) Find the equation of the line $g(x)$ that is perpendicular to $f(x)$ and passes through the point $(1/2, 0)$.

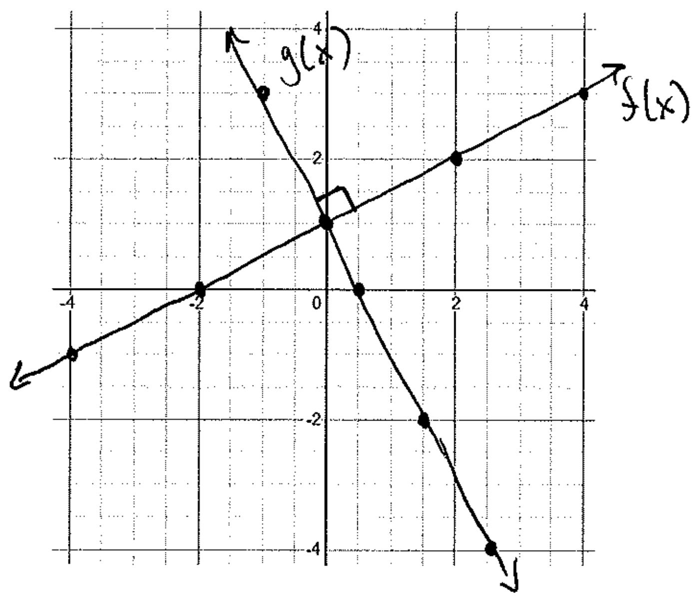
Let $g(x) = mx + b$. Since g is perpendicular to f, $m = -2$. To find b, solve $0 = g(1/2) = -2 * (\frac{1}{2}) + b$.

So $b = 1$ and $g(x) = -2x + 1$

(c) (3 points) Calculate the point where $f(x)$ and $g(x)$ intersect.

Both f(x) and g(x) have the same y-intercept. Therefore they intersect at (0,1).

(d) (3 points) Graph both lines on the grid below:

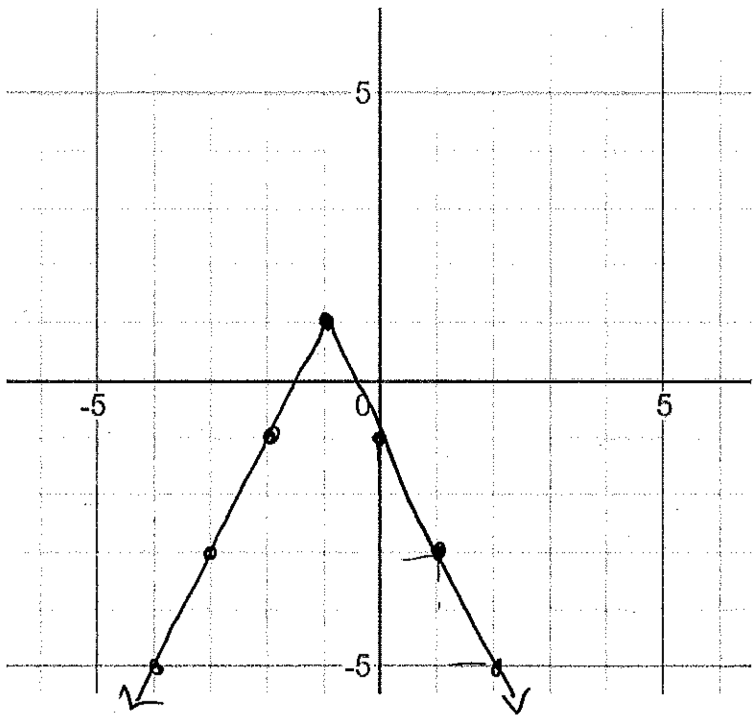


Problem 3. Let $f(x) = |x|$ and $g(x) = -2|x+1|+1$.

(a) (5 points) Explain how the graph of $g(x)$ can be obtained from the graph of $f(x)$ using transformations. Make sure to describe the transformations in the correct order!

1. Shift left by 1 unit.
2. Reflect over x-axis
3. Scale vertically by a factor of 2
4. Shift up by 1 unit

(b) (5 points) Graph $g(x)$ on the grid below.



Problem 4. Consider the quadratic function $f(x) = 3x^2 + 2x - 1$ given in general form.

(a) (3 points) Identify the vertex, the line of symmetry, and the x and y-intercepts of $f(x)$.

Vertex: $h = \frac{-2}{2 \cdot 3} = -\frac{1}{3}$, $k = f(-1/3) = 3(-1/3)^2 - \frac{2}{3} - 1$

$= \frac{1}{3} - \frac{2}{3} - \frac{3}{3} = -\frac{4}{3}$

$\Rightarrow$ vertex @ $(-1/3, -4/3)$

line of symmetry: $x = -\frac{1}{3}$

y intercept: $f(0) = -1$, y-int @ $(0,-1)$.

x intercepts: $x = \frac{-2 \pm \sqrt{4 - 4 \cdot 3 \cdot (-1)}}{2 \cdot 3} = \frac{-2 \pm \sqrt{16}}{6} = \frac{1}{3}, -1$

x-int @ $(1/3, 0)$ and $(-1, 0)$

(b) (3 points) Identify the range of $f(x)$.

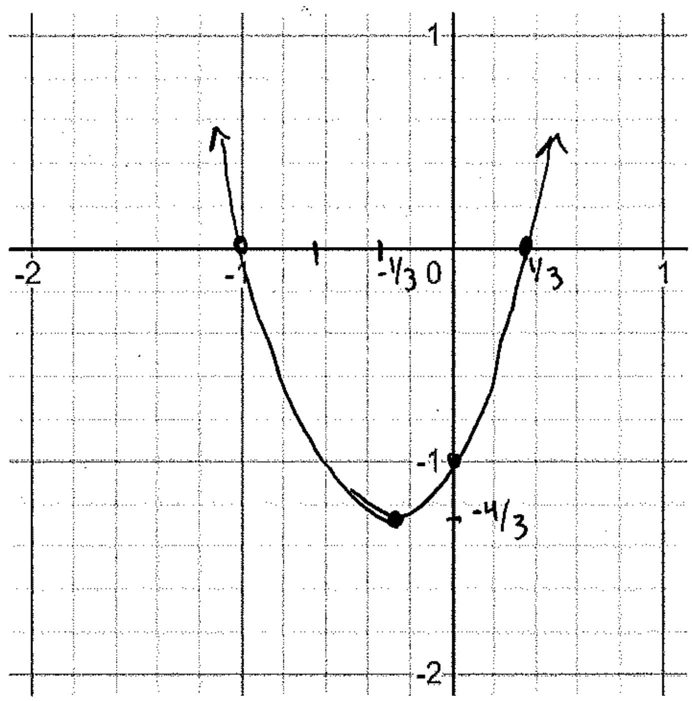
Since $a = 3 > 0$, parabola opens up and vertex is a global minimum. Therefore,

$\text{Range}(f) = [-\frac{4}{3}, \infty)$

(c) (3 points) Write $f(x)$ in standard form.

$$f(x) = 3\left(x+\tfrac{1}{3}\right)^2 - \tfrac{4}{3}$$

(d) (3 points) Graph $f(x)$ on the grid below:



Problem 5. Let $f(x) = -\frac{32}{27}(x-1)^3(x+1)$.

(a) (5 points) Identify the degree, the end-behaviour, the zeros and their multiplicities, and the y-intercept of $f(x)$.

Leading term: $-\frac{32}{27}x^4$

Degree: 4

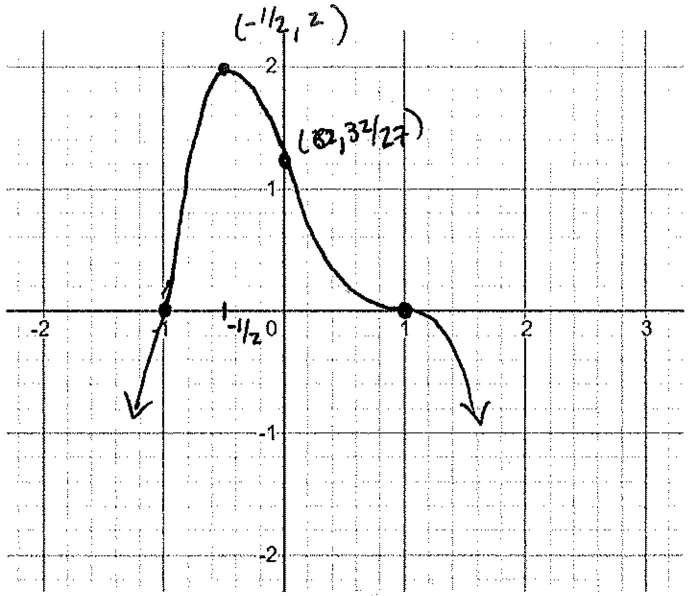
End Behaviour: $f(x) \to -\infty$ as $x \to \pm\infty$

Zeros: $x=1$, multiplicity 3

$x=-1$, multiplicity 1

y-int: $f(0) = -\frac{32}{27}(-1)^3 = \frac{32}{27} \Rightarrow (0, 32/27)$.

(b) (5 points) Graph $f(x)$ on the grid below. Make use the following fact: the point $(-1/2, 2)$ is a turning point for $f(x)$. You may use the approximation $\frac{32}{27} \approx 1.2$.



Problem 6. Ferrell's Donuts in Santa Cruz paid \$25,000 in rent, insurance, and other operating expenses in April of 2021. It costs \$.50 to produce each donut.

(a) (5 points) Find a linear model $C(d)$ that represents the cost of operating Ferell's donuts in April 2021 as a function of d the number of donuts produced.

$C(d) = 25000 + .5d$

(b) (5 points) The revenue in April 2021 was found to be given by the function $R(d) = 3d$. How many donuts did Ferell's donuts need to produce that month in order to make a profit?

In order to make a profit, revenue must exceed cost, i.e. $R(d) > C(d)$. Find the point of intersection:

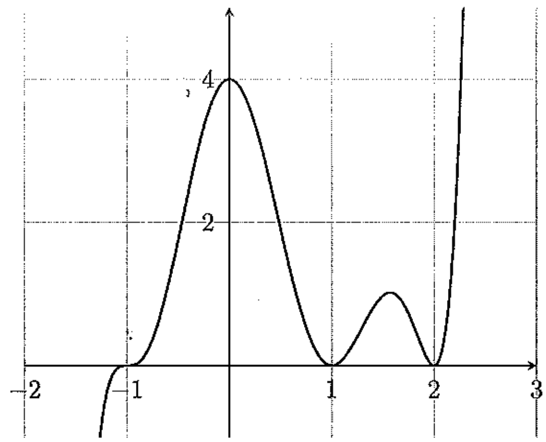
$$3d = R(d) = C(d) = 25000 + .5d$$

$\Rightarrow d = 10000$

Since both functions have positive slope, and the revenue has a larger rate of change than cost, $R(d) > C(d)$ when $d > 10{,}000$.

$\Rightarrow$ Ferrell needs to sell more than 10,000 donuts to make a profit.

Problem 7. (12 points) Consider the following graph of a polynomial.



Identify the degree, the end-behaviour, the zeros and their multiplicities, and the y-intercept. Write down an equation of smallest degree for this polynomial.

End Behaviour: $f(x) \to \infty$ as $x \to \infty$, $f(x) \to -\infty$ as $x \to -\infty$

$\Rightarrow$ Degree is odd

Zeros: $x=-1$, odd degree ≥ 3
$x=1$, even degree ≥ 2
$x=2$, even degree ≥ 2
$\Rightarrow$ degree is atleast 7

y int: $f(0)=4$.

From the above, $f(x) = a(x+1)^3(x-1)^2(x-2)^2$ for some constant a. To find a, solve

$$4 = f(0) = a \cdot 1^3 \cdot (-1)^2 \cdot (-2)^2$$

$\Rightarrow a = 1$

$\Rightarrow f(x) = (x+1)^3(x-1)^2(x-2)^2$.